

# Novel Regime for Antenna Array Oscillators Based on Exceptional Point of Degeneracy

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**Abstract**—We demonstrate a new regime of operation to conceive radiating array oscillators. This regime is based on the dispersion engineering of coupled transmission lines (CTLs) utilizing an exceptional point of degeneracy (EPD), which represents the coalescence of multiple eigenmodes. We propose the “gain and loss balance” regime for structures exhibiting significant radiation losses to enable an innovative regime for a class of coherent EPD-based radiating oscillators with stable oscillation frequency. Moreover, this class of radiating oscillators shows an interesting trend of how the oscillation threshold scales with the length of the structure. This EPD concept has potential applications in high power-efficiency oscillators and high-power radiation.

**Index Terms**—antenna array, dispersion engineering, coupled transmission lines.

## I. INTRODUCTION

In general, electromagnetic guiding structures are described by their evolution equations in terms of eigenvalues and eigenvectors (eigenmodes). We explore a very special feature of the evolution of these eigenmodes, where two or more of the supported eigenmodes of such system coalesce into a single degenerate eigenmode [1], [2]. The points in the parameter space where eigenmodes coalesce are called exceptional points of degeneracy (EPDs). The number of coalescing eigenmodes defines the order of the EPD. We study EPDs occurring in periodic guiding structures whose wave evolution are modeled by two periodic coupled TLs like the one shown in Fig. 1(a). In particular we are interested in EPDs of order higher than two. Actually, a second order EPD is naturally found in any periodic waveguide at the band edge that it is known as “regular band edge”. Here instead we explore a fourth order EPD that occurs when four independent Floquet-Bloch eigenvectors coalesce and form one single degenerate eigenvector [1], [3] at the band edge, called degenerate band edge (DBE). The DBE occurs rigorously in lossless structures as shown in [1] whereas losses and fabrication tolerances deteriorate such EPD [4]. One way to realize an EPD in lossy coupled transmission lines (CTL) is the balanced gain and loss regime [2], [4]. Here “losses” represent radiation losses in an open structure and such a scheme is useful to realize a new class of oscillatory radiating arrays based on EPDs when also gain is introduced in a proper manner.

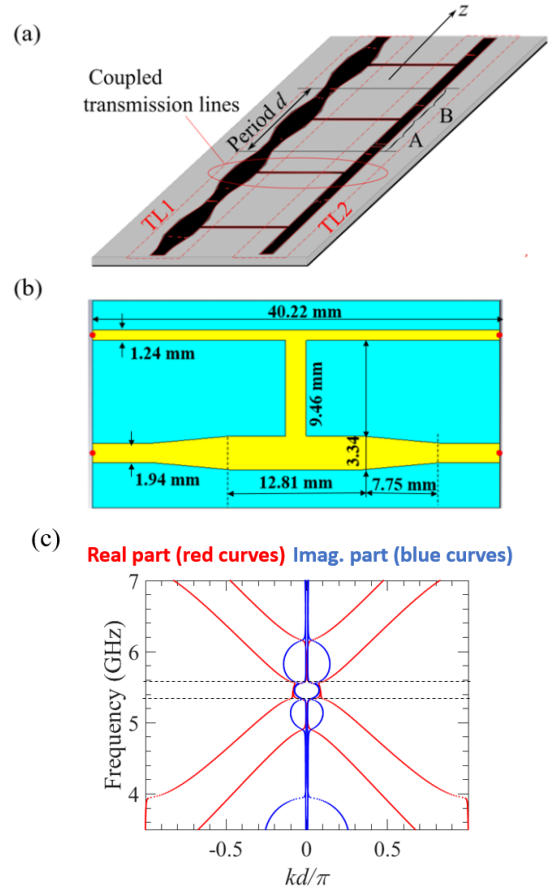


Fig. 1. (a) Schematic of two coupled microstrip lines on a grounded dielectric slab that potentially support a fourth order EPD visible in the  $(k-\omega)$  dispersion diagram at microwave frequencies. (b) Microstrip unit cell of the periodic structure that exhibits a fourth order EPD. All dimensions are in millimeters. (c) Real and imaginary parts of the wavenumber of the four guided Floquet-Bloch modes obtained using the full-wave finite element method (CST Microwave studio) accounting also for radiation, ohmic and dielectric losses. The results show that 4<sup>th</sup> order DBEs occur at 2 different frequencies (5.3 and 5.6 GHz) in the range shown in this plot. In the case analyzed, radiation losses in each unit cell are not high since near the DBE all the wavenumbers are almost real. (The non-vanishing imaginary part at the DBE would be evident in a zoomed-in plot.)

Indeed, only a very precise combination of radiation “losses” and distributed gain would enable an EPD in a radiating array which leads to the very special electromagnetic and electronic performance associated to it.

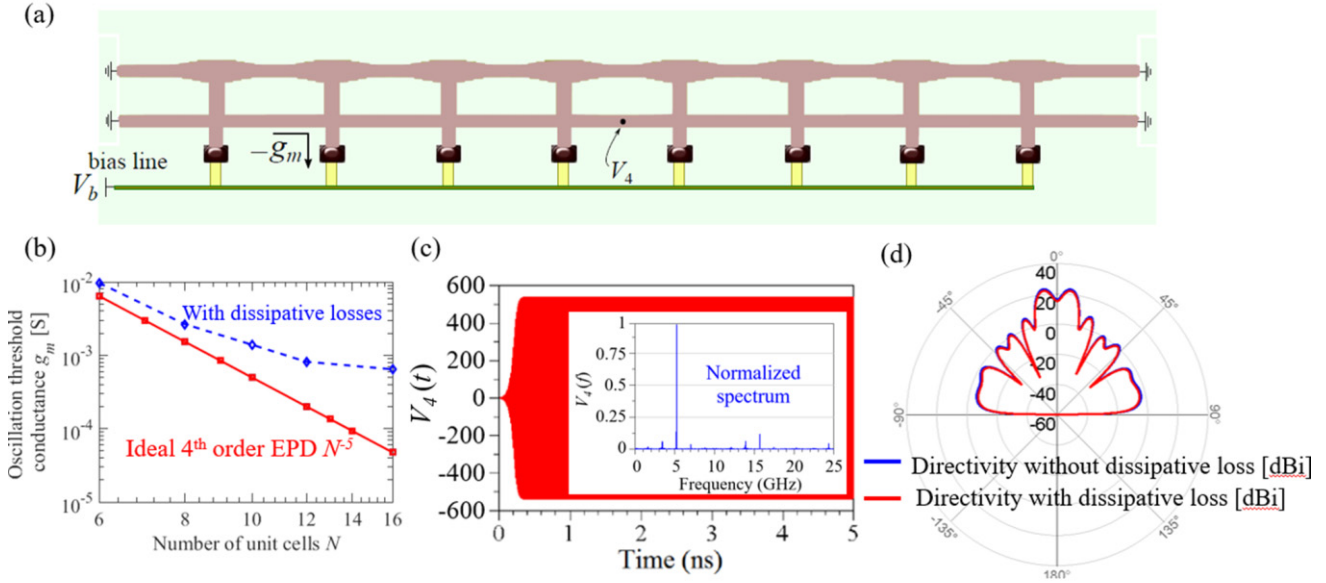


Fig. 2. (a) A microstrip-based CTL working at a gain and radiation loss balance regime made of a finite number of unit cells. Each cell has a gain lumped made of a negative conductance  $-g_m$ . The linear array is terminated in short circuits and oscillates for sufficiently large  $g_m$ . (b) Minimum gain conductance  $g_m$  (threshold) needed to start oscillations for the linear array, varying the number of unit cells. The 4<sup>th</sup> order EPD shows the  $N^{-5}$  trend while due to radiation and dissipation mechanism this trend is ceased for larger  $N$ . (c) Steady state oscillation voltage  $V_4(t)$  at the center of the array reaches saturation in less than 1 ns. The normalized spectrum of  $V_4$ , shown in inset figure, shows that oscillations occur at 5.25 GHz, which indicates that the oscillation is at a single frequency in proximity of the DBE one in Fig. 1. The spectrum is calculated by applying the Fourier transform in a time window from 2 to 5 ns. (d) Resulting broadside radiation pattern for the array oscillator with and without considering dissipative loss.

## II. SYSTEM DESCRIPTION: 4<sup>TH</sup> ORDER DBE

An EPD is a point in the parameter space of electromagnetic guiding structure where two or more eigenmodes coalesce, which means that both eigenvectors and eigenvalues coalesce. This is a stronger condition than what the term “degeneracy” usually refers to, i.e., to the coincidence of eigenvalues (wavenumbers) for different eigenvectors (field polarizations). The eigenvector degeneracy is a *necessary* condition for an EPD, where the eigenvectors are associated to the polarization states of a multimode waveguide. The importance of DBEs arises from the higher order dispersion associated to these points in lossless waveguides which possess unique physical properties [5], [6]. Here we focus on such a 4<sup>th</sup> order DBE that occurs in lossless periodic CTLs at a specific angular frequency  $\omega_e$ . The periodic unit cell dimensions are reported in Fig. 1(b). Dielectric and conductor losses are considered in our simulation results: copper with conductivity  $5.7 \times 10^7$  S/m is used for microstrip lines and ground plane, while the substrate’s relative dielectric constant is 2.2 with a loss tangent of 0.002. The substrate height is 0.508 mm. We verify the existence of the DBE in the proposed microstrip structure in Fig. 1(a) using full-wave simulations. The dispersion shown in Fig. 1(c) is carried out using full-wave simulations of the S-parameters performed using CST microwave studio based on the finite element method (FEM). The dispersion shows two frequencies (5.3 and 5.6 GHz) at which all the four wavenumbers are very close in their values to each other

denoting the occurrence of the 4<sup>th</sup> order DBE. Even though mathematically speaking the DBE is not verified exactly because of radiation losses, in practical terms the DBE prominent features are well preserved. Importantly, the DBE occurring at  $kd = 0$  is automatically associated to broadside radiation from this linear array because a Floquet harmonic has phase velocity faster than the speed of light. Modes with such spatial harmonic are leaky waves.

## III. ANTENNA ARRAY OSCILLATOR USING GAIN AND LOSS BALANCED EPD

We demonstrate here our proposed scheme of operation to conceive a new class of distributed single-frequency radiating array oscillators. Our scheme is based on utilizing a fourth order EPD with “gain and loss balance” that can be used to improve the performance of active integrated antennas [9] and grid oscillators [10]. To this aim we utilize the passive radiating CTL shown in Fig. 1 where the DBE is slightly perturbed by radiation losses and introduce gain to the system to have balance condition. This condition is achieved by incorporating an active component (i.e. gain) in each unit cell to balance the radiation loss and hence generate oscillations. Gain here is considered as a *lumped* negative conductance namely  $-g_m$  ( $g_m$  has a positive value in Siemens [S] and is assumed to be the same in each unit cell), located as seen in Fig. 2(a). The negative conductance considered here can be practically implemented by transistors, and we use a simple model consisting of the third-order  $I$ - $V$

characteristic as  $I(t) = -g_m V(t) + \alpha V^3(t)$ , where  $-g_m$  is the slope of the  $I$ - $V$  curve in the negative resistance region [11], and  $\alpha$  is the third-order nonlinearity constant that models the saturation characteristic of the device. Calculations for the structure in Fig. 2(a) including radiation and dissipative losses are carried out using the time domain transient solver in Keysight Technologies ADS. The oscillating EPD array shows a steady state oscillation that has a single frequency of 5.25 GHz shown in Fig. 2(c) almost coinciding with the DBE frequency shown in Fig. 1(c). The waveform of the voltage at the center of the array  $V_4(t)$  reaches steady state in less than 1 ns. The oscillation frequency is determined by obtaining the spectrum of  $V_4(t)$  in a time window from 2 to 5 ns, which is shown in the inset of Fig. 2(c) confirming the single frequency oscillation regime. In Fig. 2(b), we plot the oscillation threshold  $g_{m,th}$  which is the minimum value of the gain conductance  $g_m$  (in each unit cell) to start oscillations in the finite-length structure. The trend of  $g_{m,th}$  versus number of unit cells shows the fundamental scaling feature of the DBE oscillator versus cavity length as  $g_{m,th} \propto N^{-5}$ .

In Fig. 2(d), we present the radiation pattern calculated via MoM full-wave simulations in Keysight Technologies ADS at the steady state fundamental frequency depicted in Fig. 2(c), showing that the linear array is mostly radiating close to the broadside direction as expected, since we use the DBE associated to a periodic-cell with phase shift with  $kd = 0$ .

#### IV. CONCLUSION

We have demonstrated a fourth order DBE in microstrip CTLs at microwave frequencies through full-wave simulations. Importantly, we have demonstrated a new paradigm for radiating array oscillators developed by coupled transmission lines utilizing DBEs. The new regime of operation of such radiating array oscillator is based on the DBE with a gain and loss balance regime, where losses (seen from a circuit point of view) are mostly due to radiation, and gain is realized by incorporating active elements in each periodic unit cell. This led to a single frequency radiating oscillator with a broadside radiation that shows unique scaling of its oscillation threshold with the array length.

This new operational scheme would be valuable in enhancing the efficiency of spatial power combiners and active antennas and various devices requiring coherent emission at microwaves and millimeter waves.

Despite the case shown in this paper where each unit cell does not exhibit very high “radiation losses” (i.e., a small radiation leakage is occurring in each unit cell), the same scheme can be adopted to radiating arrays where each unit cell exhibits very high radiation losses. In such a case large values of gain must be used in each unit cell, with possible high-power efficiency.

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